

Non-Markovian Entanglement Sudden Death and Rebirth of a Two-Qubit System in the Presence of System-Bath Coherence

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Abstract

We present a detailed study of the entanglement dynamics of a two-qubit system coupled to independent non-Markovian environments, employing hierarchy equations. This recently developed theoretical treatment can conveniently solve non-Markovian problems and take into consideration the correlation between the system and bath in an initial state. We concentrate on calculating the death and rebirth time points of the entanglement to obtain a general view of the concurrence curve and explore the behavior of entanglement dynamics with respect to the coupling strength, the characteristic frequency of the noise bath and the environment temperature.

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I. INTRODUCTION

Quantum information, including quantum entanglement and quantum dissonance [1–3], has been a popular area of research in recent years. In this field, entanglement undoubtedly plays a central role as an important resource in quantum computation [4], teleportation [5], dense coding [6] and cryptography [7, 8]. There have been many meaningful works on the dynamics of entanglement in multiple qubits, especially two qubits, interacting with different kinds of environments and we now know some important features of the entanglement such as the entanglement sudden death [9–12] and birth [13].

However, environments with different properties will have different effects on the entanglement dynamics. Recently quantum systems in a non-Markovian environment have been a subject of great interest [2, 14–19]. Theoretical treatments have been developed to deal with this situation, such as Ref. [15], [16] and [20]. Of particular interest to us is the hierarchy equations approach employed by Dijkstra and Tanimura [14]. This treatment is deduced using the influence functional method of Feynman and Vernon without the limitation of perturbative, Markovian or rotating wave approximations. It can easily take into account the effect of the system-bath coupling on the dynamics of the entanglement for any initial conditions. In their work, they discussed the entanglement evolution of two qubits interacting with a quantum-mechanical bath and then compared this hierarchy method with the Redfield equation. It is found that the result of the full calculation markedly differs from the Redfield predictions. In the present work we continue this meaningful work.

We first introduce the method of hierarchy equations of motion developed by Y. Tanimura and coworkers in Ref. [21–23]. Then we employ this method to carefully calculate the entanglement dynamics of a two-qubit system interacting with a non-Markovian environment, determining the influence of the strength of the system-bath interaction, the characteristic frequency of the bath and the environment temperature on the time evolution of the entanglement, especially on the sudden death and sudden birth time points.

The entanglement of the two qubits should be measured using Wootters’ concurrence [24]:

$$\mathcal{C}(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (1)$$

and

$$\rho = \rho_{AB}(\sigma_y^A \otimes \sigma_y^B) \rho_{AB}^* (\sigma_y^A \otimes \sigma_y^B), \quad (2)$$

where ρ_{AB} is the density matrix of system AB, ρ_{AB}^* denotes the complex conjugation of ρ_{AB} and $\sigma_y^{A(B)}$ is the Pauli matrix. λ_i are the eigenvalues of ρ (λ_1 should be the largest eigenvalue).

II. THE MODEL AND THEORY

For simplicity, we set $\hbar = 1$ throughout this report. We assume that the two qubits are coupled independently to two identical baths with the same strength. The baths have a characteristic frequency γ , and large and small γ indicate fast and slow baths respectively. The energy gap of the qubits is ε and the qubits are coupled by an interaction ζ . Therefore, we can write the standard system Hamilton as [14, 25]

$$H_S = \varepsilon(a_1^\dagger a_1 + a_2^\dagger a_2) + \zeta(a_1^\dagger + a_1)(a_2^\dagger + a_2). \quad (3)$$

The subscripts 1 and 2 represent the two qubits and a^\dagger and a are the creation and annihilation operators. We choose a model that adequately represents the environment as a set of oscillators with a coupling linear in the oscillator coordinates, having the form [26]

$$H_B = \sum_j \left(\frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \right) \quad (4)$$

and

$$H_{SB} = - \sum_{m,j} C_{mj} (a_m^\dagger + a_m) x_j, \quad (5)$$

where H_B is the Hamilton for the bath and H_{SB} is the interaction item of the system and bath. x_j , p_j , m_j , and ω_j are the coordinate, momentum, mass, and frequency of the j th harmonic oscillator, respectively. C_{mj} is the strength of coupling of the m th qubit to the j th oscillator. To obtain complete information about the effect of the environment, we introduce the spectral density $J_m(\omega)$, which is defined as

$$J_m(\omega) = \frac{\pi}{2} \sum_{j=1} (C_{mj}^2 / m_j \omega_j) \delta(\omega - \omega_j), \quad (6)$$

and in this report we assume that

$$J_m(\omega) = \omega\eta\gamma/(\omega^2 + \gamma^2) \quad (7)$$

which comes from the Lorentzian cutoff [22], and is the same for both qubits. The hierarchy equations approach then gives the equation of motion for the system density matrix, which has the form [14, 23, 25]:

$$\begin{aligned} \frac{d\rho_{\mathbf{n}}(t)}{dt} = & -(i\mathcal{L} + \sum_{m=1}^2 \sum_{k=0}^K n_{mk}\gamma_k)\rho_{\mathbf{n}}(t) - \sum_{m=1}^2 \left(\left(\frac{1}{\beta\gamma_0} - i\frac{1}{2} \right) \eta - \sum_{k=0}^K \frac{c_k}{\gamma_k} \right) [V_m, [V_m, \rho_{\mathbf{n}}(t)]] \\ & - i \sum_{m=1}^2 \sum_{k=0}^K [V_m, \rho_{n_{mk}+1}(t)] - i \sum_{m=1}^2 \sum_{k=0}^K n_{mk} (c_k V_m \rho_{n_{mk}-1}(t) - c_k^* \rho_{n_{mk}-1}(t) V_m), \quad (8) \end{aligned}$$

where $\mathcal{L}\rho = [H_S, \rho]$, $\gamma_0 = \gamma$ (the bath frequency), $\gamma_k = 2\pi k/\beta$ (for $k \geq 1$) are Matsubara frequencies, $c_0 = \frac{\eta\gamma}{2}(-i + \cot \beta\gamma/2)$, $c_k = 2\eta\gamma_0\gamma_k/\beta(\gamma_k^2 - \gamma_0^2)$, $k \geq 1$, and $V_m = a_m^\dagger + a_m$.

In this equation, only $\rho_{\mathbf{0}}$ represents the physical system density operator, and other $\rho_{\mathbf{n}}$ are called auxiliary density operators (ADOs). The subscript \mathbf{n} is a multi-index, which has $2 \times (K+1)$ dimensions and can be extended as n_{mk} , i.e. $n_{10}, n_{11}, \dots, n_{1K}; n_{20}, n_{21}, \dots, n_{2K}$. The notation $n_{mk} \pm 1$ refers to an increase and decrease of this index. When the auxiliary density operators are all zero, the system and bath are not coupled; when the density operators are nonzero, we should properly take the coupling into consideration, which will influence the entanglement evolution dramatically. All of these values contain important information about the coupling.

The method can be employed to calculate the dynamics of a system in a bath from any initial state (correlated or not) in a computer program with the truncating method developed by A. Ishizaki and Y. Tanimura [23]. In the present work, we investigate the behavior of the entanglement death and rebirth time points with respect to several influential factors mentioned above. However, the range of all parameters should be carefully determined so as to obey the termination approximation and ensure that the results are sufficiently precise.

III. THE RESULT AND DISCUSSION

We choose $\varepsilon = 1.5\zeta$, η from 0.4ζ to 0.8ζ , γ from 0.4ζ to 1.1ζ , and $\beta\zeta$ from 2 to 3. Our initial state is chosen as

$$\rho_0(0) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & -0.5 & 0 \\ 0 & -0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (9)$$

in the standard product $|e_1e_2\rangle$, $|e_1g_2\rangle$, $|g_1e_2\rangle$, $|g_1g_2\rangle$, and e_m and g_m are the excited state and ground state of the m th qubit.

A. System-Bath Interaction

We know that η is related to the system-bath coupling strength. Though the function of η in the entanglement dynamics we can infer that when the coupling is weak, a slow decay of the concurrence is expected because the flow of information from the system into the bath will be slow, whereas the decay will definitely be rapid when the system and bath are tightly correlated. Figure 1 proves our inference; we see that the decay of concurrence is slower when η is small than when η is large. The figure also indicates that small η may result in strong vibration of the concurrence. The environment can return a small amount of information to the system during the evolution. With weak coupling (small η), this little returned

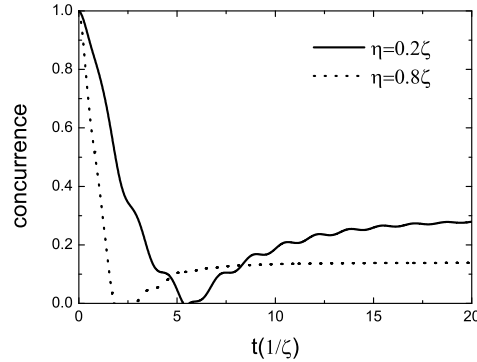


FIG. 1: Concurrence as a function of time with $\beta\zeta = 2.5$, $\gamma = 0.5\zeta$, $\eta = 0.2\zeta$ for the black line and $\eta = 0.8\zeta$ for the red line.

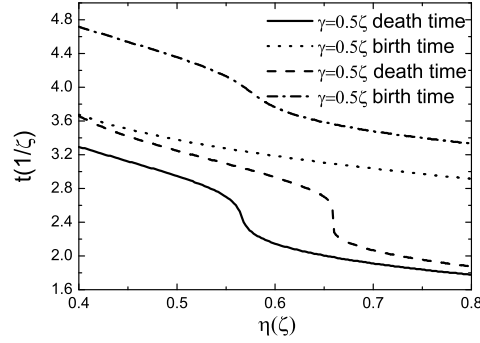


FIG. 2: Entanglement death time and rebirth time as functions of η , with $\gamma = 0.5\zeta$ for the solid and dotted lines and $\gamma = 0.4\zeta$ for the dashed and dash-dotted lines, $\beta\zeta = 2.5$.

information will have a strong effect on the curve of the entanglement when the concurrence is small, and thus, there is oscillation. As η increases, the curve becomes smoother, and there is only one pair of death and birth time points. From Fig. 2, we see clearly the change in death and birth time points with the increase in η . The negative slopes of these curves are consistent with the effect of η discussed above. It is interesting to find that at some intervals the slopes of these curves suddenly change in Fig. 2. This phenomenon is due to a change in the slope near the death and birth points on the concurrence curve.

B. Characteristic Frequency of the Bath

Figures 3 and 4 show the influence of the characteristic frequency of the bath on the dynamics of the entanglement. Large γ means a fast bath. Because the frequency of the two level system in this report is 1.5ζ , and if we take into account the range of γ discussed here we know that the bath is slow. Most theoretical treatments with which we describe this entanglement dynamics are valid only if there is a fast bath, which has a much larger characteristic frequency than the system. Only in the fast bath scenario can we choose the initial state in which the system and bath are not correlated [14]. However, as mentioned before, the hierarchy method can easily solve a slow bath situation, taking into consideration the bath effect on the entanglement in the initial state. A slow bath can receive and return the information from the system slowly and a fast bath may absorb all the information in a very short time and there may not even be revival, as we show in Fig. 3. This can explain

the delay in the death and birth when γ is small. More details are shown in Fig. 4. The death and birth time points approach zero with an increase in γ . With smaller η , we see that all time points increase, for the reason discussed above.

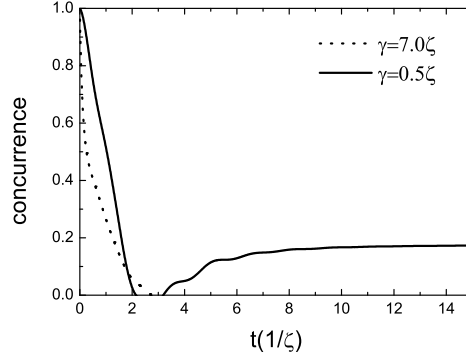


FIG. 3: Concurrence as a function of time with $\beta\zeta = 2.5$, $\eta = 0.6\zeta$, $\gamma = 7\zeta$ for the dotted line and $\gamma = 0.5\zeta$ for the solid line.

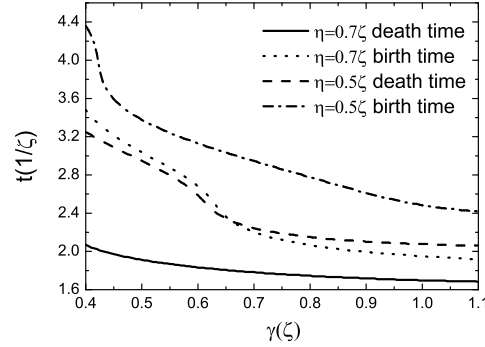


FIG. 4: Entanglement death time and rebirth time as functions of γ , with $\eta = 0.5\zeta$ for the dash and dash-dotted lines and $\eta = 0.7\zeta$ for the solid and dotted lines, $\beta\zeta = 2.5$.

C. Temperature

Temperature influences the system dramatically. In Ref.[14] it is seen that the low-temperature scenario is unsuitable for Born and ultrafast bath approximations, which are used in many theoretical treatments; however, this scenario is in the solvable range of the hierarchy equations approach. In Fig. 5, the death time point increases and birth time

point decreases with a decrease in temperature. That is to say, the entanglement revives more quickly at lower temperature. The figure infers that the concurrence curve should be sufficiently smooth, and with the increase in $\beta\zeta$, the whole curve is raised up to obtain a smaller death time interval. If we simulate the two curves in the figure and calculate the relationship between $\beta\zeta$ and the time point, then we may simply change the environment temperature to prepare our desired states and dynamics of the entanglement in experiments.

In fact, with higher hierarchy of the ADOs, we can investigate a larger $\beta\zeta$ scenario. However, the computation time is too great as it increases exponentially with higher orders.

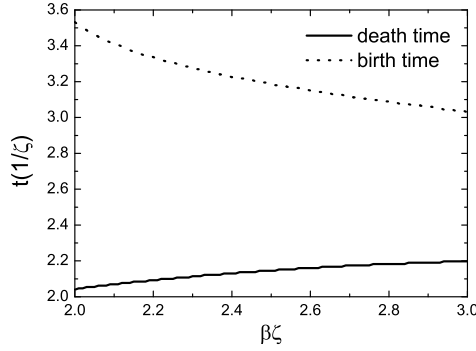


FIG. 5: Sudden death time point (solid line) and sudden birth time point (dotted line) of entanglement with different values of $\beta\zeta$ from 2 to 3. Other parameters are $\eta = 0.6\zeta, \gamma = 0.5\zeta$.

IV. CONCLUSION

In this report we studied the entanglement dynamics of two-qubit in a non-Markovian environment using the recently developed hierarchy equations approach. We explored in detail the role of parameters in the entanglement evolution, including the system-bath interaction η , the temperature β and the bath frequency γ . All these important features are useful when preparing a desired system state in an experiment. We also showed this hierarchy equations approach to be effective in calculating the dynamics of a system in many possible scenarios. Applying a higher order of the equation for a broader range of parameters and the physical meaning of non-zero ADOs will be future work.

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